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Answers and Hints

13.3 Introductory Offers

If a high quality firm sells to a consumer in the first period at some price p_1, then in the second period the consumer will be willing to pay $p_2 = h$, since he knows the product is high quality. Knowing that it can make a profit $h - c_h$ from a customer in the second period, a high quality firm might want to make a consumer an 'introductory offer' at a price p_1 in the first period that would not be mimicked by the low quality firm, in order to reap the second period profit.

If $p_1 > c_l$, the low quality firm could mimic the high quality firm, so the best the high quality firm can do is to charge $p_1 = c_l$, which the low quality firm will not mimic, since the low quality firm cannot profit by doing so (it cannot profit in the first period, and the consumer will not buy the low quality product in the second period). In this case the high quality firm's profits are $(c_1 - c_h) + \delta(h - c_h)$ As long as these profits are positive, which reduces to $h > c_h + \delta(c_h - c_l)$, the high quality firm will stay in business.

13.4 Web Sites (for Spiders!)

(a) In a truthful signalling equilibrium, strong spiders use *saa* and weak spiders use *wwa*. To see this, note that strong spiders say they're strong and weak spiders say they're weak, by definition of a truthful signalling equilibrium. Weak spiders withdraw against strong spiders because $d > 0$, and attack other weak spiders because $v - c_w > 0$. Strong spiders attack weak spiders if they do not withdraw, because $2v - b > 2v - c_s > v$.

(b) If p is the fraction of strong spiders, then the expected payoff to a strong spider is $p(v - c_s) + 2(1 - p)v - e$, and the expected payoff

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to a weak spider is $(1-p)(v-c_w)$. If these two are equal, then

$$p = \frac{v + c_w - e}{c_w + c_s}, \tag{13.1}$$

which is strictly between 0 and 1 if and only if $e - c_w < v < e + c_s$.

(c) In a truthful signalling equilibrium, each spider has expected payoff

$$\pi = \frac{(v - c_w)(c_s + e - v)}{c_w + c_s}. \tag{13.2}$$

Suppose a weak spider signals that it is strong, and all other spiders play the truthful signalling equilibrium strategy. If the other spider is strong, it will attack and the weak spider will receive $-d$. If the other spider is weak it will withdraw, and the spider will gain $2v$ Thus the payoff to the spider for a misleading communication is $-pd+2(1-p)v$, which cannot be greater than (13.2) if truth-telling is Nash. Solving for d, we get

$$d \geq \frac{(c_s + e - v)(v + c_w)}{c_w - e + v}.$$

Can a strong spider benefit from signaling that it is weak? To see that it cannot, suppose first that it faces a strong spider. If it attacks the strong spider after signaling that it is weak, it gets the same payoff as if it signalled strong (since its opponent always attacks). If it withdraws against its opponent, it gets 0, which is less than the $v-c$ it gets by attacking. Thus signaling weak against a strong opponent can't lead to a gain. Suppose the opponent is weak. Then signaling weak means that the opponent will attack. Responding by withdrawing, it gets 0; responding by attacking, it gets $2v-b$, since it always defeats its weak opponent. But if it had signalled strong, it would have earned $2v > 2v - b$. Thus it never pays a strong spider to signal that it is weak.

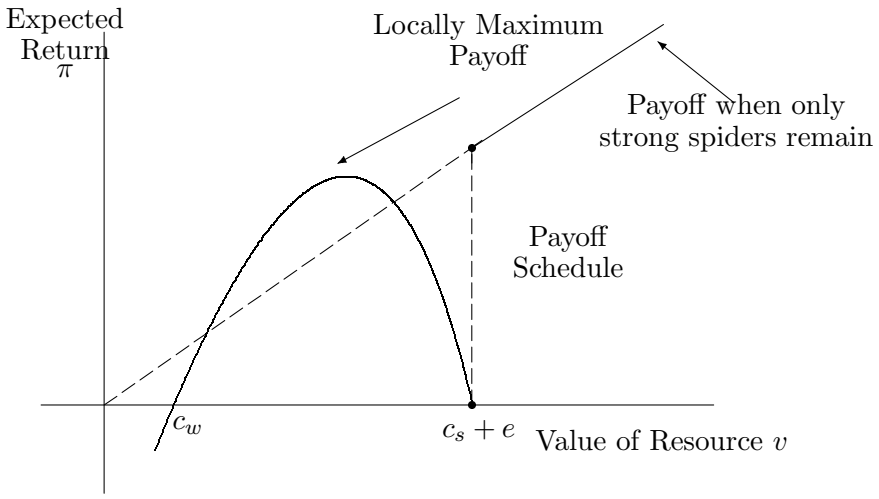
(d) This result follows directly from equation (13.2). This result occurs because higher e entails a lower fraction of strong spiders, from (13.1). But weak spiders earn $(1-p)(v-c_w)$, which is decreasing in p, and strong spiders earn the same as weak spiders in equilibrium.

(e) We differentiate (13.2) with respect to v, getting

$$\pi_v = \frac{c_w + c_s + e - 2v}{c_w + c_s}.$$

When $v = e - c_w > 0$, its smallest value, $\pi_v = 3c_w + c_s - e$, has indeterminate sign. If it is negative, lowering v would increase payoff, as asserted. So suppose $\pi_v > 0$ at $v = e - c_w$. When $v = e + c_s$, its largest value, $\pi_v = c_w - c_s - e$, which is negative. Thus there is an interior optimum for v in general.

Notice that this optimum is only local; if we increase v beyond $e + c_s$ only strong spiders remain, and their payoff is now strictly increasing in v. Here is a picture, showing the discontinuity in the payoffs, once there are no weak spiders left. At this point the strong spiders no longer signal or attack, since all know all are strong. They each get v in this case.



13.7 The Shepherds who Never Cry Wolf

The following payoffs are easy to derive:

$$\pi_1(N,N) = p(1-a) + (1-p)(1-b); \quad \pi_2(N,N) = 1;$$
$$\pi_1(N,H) = p(1-a) + (1-p)(1-b); \quad \pi_2(N,H) = 1;$$
$$\pi_1(N,A) = 1; \quad \pi_2(N,A) = 1-d;$$

$$\pi_1(H,N) = p(1-a) + (1-p)(1-b) - pc; \quad \pi_2(H,N) = 1;$$
$$\pi_1(H,H) = p(1-c) + (1-p)(1-b); \quad \pi_2(H,H) = p(1-d) + 1 - p;$$
$$\pi_1(H,A) = 1 - pc; \quad \pi_2(H,A) = 1 - d;$$
$$\pi_1(A,N) = p(1-a) + (1-p)(1-b) - c; \quad \pi_2(A,N) = 1;$$
$$\pi_1(A,H) = 1 - c; \quad \pi_2(A,H) = 1 - d;$$
$$\pi_1(A,A) = 1 - c; \quad \pi_2(A,A) = 1 - d.$$

Now the total payoff for shepherd 1 is $\pi_1^t = \pi_1 + k\pi_2$, and the total payoff for shepherd 2 is $\pi_2^t = \pi_1 + k\pi_1$. Substituting in numbers and forming the normal form matrix for the game, we get

	N	H	A
N	$\frac{19}{24}, \frac{37}{32}$	$\frac{19}{24}, \frac{37}{32}$	$\frac{21}{16}, \frac{7}{6}$
H	$\frac{95}{192}, \frac{793}{768}$	$\frac{47}{48}, \frac{829}{768}$	$\frac{65}{64}, \frac{267}{256}$
A	$\frac{19}{48}, \frac{571}{576}$	$\frac{11}{12}, \frac{577}{576}$	$\frac{11}{12}, \frac{577}{576}$

It is easy to see that (H,H) and (N,A) are Nash equilibria, and you can check that there is a mixed strategy equilibrium in which the threatened shepherd uses $\frac{1}{3}N + \frac{2}{3}H$ and the other shepherd uses $\frac{3}{5}H + \frac{2}{5}A$.

13.9 Honest Signally among Partial Altruists

The payoff matrix for the encounter between a fisher observing a threatened fisher is as follows:

	Never Ask	Ask If Distressed	Always Ask
Never Help	$r(1-p)u$ $(1-p)u$	$r(1-p)u-rpt$ $(1-p)u-pt$	$r(1-p)u-rt$ $(1-p)u-t$
Help If Asked	$r(1-p)u$ $(1-p)u$	$r[p(1-t)+(1-p)u]-pc$ $p(1-t)+(1-p)u$	$r[p+(1-p)v-t]-c$ $p+(1-p)v-t$
Always Help	$r[p+(1-p)v]-c$ $p+(1-p)v$	$r[p(1-t)+(1-p)v]-c$ $p(1-t)+(1-p)v$	$r[p+(1-p)v-t]-c$ $p+(1-p)v-t$

The total payoff for the players is thus

	Never Ask	Ask If Distressed	Always Ask
Never Help	$(1+r)(1-p)u$	$(1+r)[(1-p)u-pt]$	$(1+r)[(1-p)u-t]$
Help If Asked	$(1+r)(1-p)u$	$(1+r)[p(1-t)+(1-p)u]-pc$	$(1+r)[p+(1-p)v-t]-c$
Always Help	$(1+r)[p+(1-p)v]-c$	$(1+r)[p(1-t)+(1-p)v]-c$	$(1+r)[p+(1-p)v-t]-c$

The answers to the problem can be obtained in a straightforward manner from these matrices.

13.11 Education as A Screening Device

(a) Given the probabilities (c), the wages (b) follow from

$$w_k = \mathrm{P}[a_h|e_k]a_h + \mathrm{P}[a_l|e_k]a_l, \qquad k = h, l. \tag{13.3}$$

Then it is a best response for workers to choose low education whatever their ability type, so (a) follows. Since both types choose e_l, the conditional probability $\mathrm{P}[a_l|e_l] = 1 - \alpha$ is consistent with the behavior of the agents, and since e_h is off the path of play, any conditional for $\mathrm{P}[a_l|e_h]$ is acceptable, so long as it induces a Nash equilibrium.

(b) Assume the above conditions hold, and suppose c satisfies

$$a_l(a_h - a_l) < c < a_h(a_h - a_l).$$

The wage conditions (b) follow from (13.3) and (c). Also, $a_h - c/a_h > a_l$, so a high ability worker prefers to choose $e = 1$ and signal his true type, rather than choose e_l and signal his type as low ability. Similarly, $a_l > a_h - c/a_l$, so a low ability worker prefers to choose e_l and signal his true type, rather than choose e_h and signal his type as high ability.

(c) The wage conditions (b) follow from (13.3) and (c). Suppose $c < a_l(a_h - a_l)$ Then both high and low ability workers prefer to get education and the higher wage w_h rather than signal that they are low quality.

(d) Let $\bar{e} = \alpha a_l(a_h - a_l)/c$, and choose $e^* \in [0, \bar{e}]$. Given the employer's wage offer, if a worker does not choose $e = e^*$, he might as well choose $e = 0$, since his wage in any case must be $w = a_l$. A low-ability worker then prefers to get education e^* rather than any other educational level, since

$$a_l \leq \alpha a_h + (1 - \alpha)a_l - \frac{ce^*}{a_l}.$$

This is *a fortiori* true for the high ability worker, whose incentive compatibility constraint is not binding.

(e) Consider the interval

$$\left[\frac{a_l(a_h - a_l)}{c}, \frac{a_h(a_h - a_l)}{c}\right].$$

If c is sufficiently large, this interval has a nonempty intersection with the unit interval $[0, 1]$. Suppose this intersection is $[e_{min}, e_{max}]$. Then for $e^* \in [e_{min}, e_{max}]$, a high ability worker prefers to acquire education e^* and receive the high wage $w = a_h$, while the low ability worker prefers to receive $w = a_l$ with no education.

13.12 Capital as a Signaling Device: A Model of Productivity Enhancing Redistributions

(a) Given $p > 0$, choose k so that

$$1 > k(1 + \rho) > q + p(1 - q).$$

This is possible since $q + p(1 - q) < 1$. Then it is clear that the fraction q of good projects are socially productive. The interest rate r that a producer must offer then must satisfy

$$k(1 + \rho) = qk(1 + r) + (1 - q)kp(1 + r) = k(1 + r)[q + p(1 - q)],$$

so

$$r = \frac{1 + \rho}{q + p(1 - q)} - 1. \tag{13.4}$$

The net profit of a producer with a good project is then

$$1 - k(1 + r) = \frac{q + p(1 - q) - k(1 + \rho)}{q + p(1 - q)} < 0,$$

so such producers will be unwilling to offer lenders an interest rate they are willing to accept. The same is clearly true of bad projects, so no projects get funded. Note that bad projects are not socially productive in this case, since

$$p - k(1+\rho) < p - (q + p(1-q)) = -q(1-p) < 0.$$

(b) Choose k so that

$$p < k(1+\rho) < q + p(1-q),$$

which is clearly always possible. Then the fraction $1-q$ of bad projects are socially unproductive. The interest rate r must still satisfy equation (13.4), so the payoff to a successful project (good or bad) is

$$1 - k(1+r) = \frac{q + p(1-q) - k(1+\rho)}{q + p(1-q)} > 0,$$

so producers of both good and bad projects are willing to offer interest rate r, and lenders are willing to lend at this rate to all producers.

(c) Let

$$k^p_{min} = \frac{p[1 - k(1+\rho)]}{(1-p)(1+\rho)}.$$

Note that since good projects are socially productive, $k^p_{min} > 0$. Suppose all producers have wealth $k^p > k^p_{min}$, and lenders believe that only a producer with a good project will invest k^p in his project. Then lenders will be willing to lend at interest rate ρ. Ia a producer invests k^p in his project and borrows $k - k^p$, his return is 1 and his costs are foregone earnings $k^p(1+\rho)$ and capital costs $(k-k^p)(1+\rho)$. Thus his profit is

$$1 - k^p(1+\rho) - (k - k^p)(1+\rho) = 1 - k(1+\rho) > 0,$$

so such a producer is willing to undertake this transaction. If the producer with a bad project invest his capital k^p, his return is

$$p[1 - k(1+\rho)] - (1-p)k^p(1+\rho) < 0,$$

so he will not put up the equity. This proves the theorem.